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COMPUTER PROGRAM FOR CALCULATING ISOTHERMAL, TURBULENT JET MIXING OF TWO GASES

by Leo F. Donovan and Carroll A. Todd

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Cleveland, Ohio

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SUMMARY

Fluid mechanics is perhaps the most significant area in the investigation of the coaxial-flow, gas-core nuclear reactor. This report describes a computer program for solving a simplified model of the turbulent mixing that occurs between a central fuel jet and a surrounding, faster-moving coaxial stream of propellant. As such, this report constitutes a step toward a better understanding of this aspect of the gas-core nuclear rocket. Local values of time-averaged velocity and the mass fraction of fuel can be calculated for a reactor in less than a minute on the IBM 7094.

The von Mises transformation was used to convert the axisymmetric forms of the isothermal boundary layer momentum and diffusion equations to forms amenable to numerical solution. The effects of confining walls were not considered. The program can solve problems in which the initial velocities and densities of the two streams differ greatly, by using expressions for eddy viscosity that vary radially as well as axially.

The effects of initial coaxial-stream- to jet-velocity and density ratios on velocity and mass fraction profiles are shown. An unspecified reference density occurs in the eddy viscosity formulation; the influences on velocity and mass fraction profiles of several choices for the reference density are illustrated. Radial and axial variations of eddy viscosity and the product of density and eddy viscosity are also given. Estimates are made of the effects of initial velocity ratio and initial density ratio on the mass of the major jet component contained within a given volume. The maximum and minimum amounts that could be present are included for comparison.

INTRODUCTION

The concept of a coaxial-flow, gaseous nuclear reactor provides the motivation for the work to be described in this report. A brief discussion of some of the features of this concept will be helpful in understanding the relevance of turbulent jet mixing.

An important characteristic of rocket performance is specific impulse; specific impulse is the thrust obtained per unit weight flow rate of propellant mixture expelled from the engine. High specific impulse is desirable since less propellant will be required to produce a given total impulse (i.e., integral of thrust over the operating time of the reactor). Specific impulse is approximately proportional to the square root of the ratio of exhaust temperature to molecular weight; thus, high temperatures and low molecular weights are desirable. Chemical rocket performance is limited by the temperature to which the heat of combustion will raise a given fuel-oxidizer combination; for example, advanced hydrogen-oxygen rockets produce a specific impulse of about 450 seconds. The great advantage of nuclear rockets is the high specific impulse that can be obtained by using hydrogen as the propellant. Solid-core nuclear reactors, however, must operate at temperatures that fuel-bearing materials can withstand and are thus limited to specific impulses of about 1000 seconds.

Higher fuel temperatures are possible in the gas-core nuclear reactor since the fuel is not supported on solid surfaces. Rather, a slowly moving gaseous fuel mass radiates heat to a coflowing annular propellant stream. The initial ratio of propellant to fuel velocity must be high in order to keep the loss of fuel as low as possible. With such a reactor, specific impulses of 2000 to 3000 seconds may be possible. Use of nuclear fuel and hydrogen propellant results in small initial propellant- to fuel-density ratios.

The gas-core nuclear reactor problem was made more amenable to solution by dividing it into three major areas (ref. 1) (viz, nuclear aspects, radiant heat transfer, and fluid mechanics). The goal is to recombine the separate parts into a meaningful whole after each relevant process is understood. The first two areas have been discussed in part elsewhere (refs. 2 and 3); fluid mechanics, including mass transfer, was considered in reference 4 and is treated in the present report. In reference 4 molecular transport coefficients were retained, and eddy viscosity was assumed to be a function of axial position raised to an arbitrary power which was determined by comparison with data from a bromine-air experiment.

In the present analysis a modified Prandtl eddy viscosity was used along with the fact that in turbulent jet mixing the molecular transport coefficients are negligible compared with the turbulent transport coefficients. The equations governing the mixing are the continuity equation, the momentum equation, and the diffusion equation. Boundary layer assumptions were used to simplify these equations. A sketch of the model analyzed is shown in figure 1. The problem at hand differs from what has been solved before in that a large initial coaxial-stream- to jet-velocity ratio is coupled with a small coaxial-stream- to jet-density ratio, so that an appropriate formulation of the eddy viscosity is not known.

The problem was simplified for this report by eliminating the effect of confining walls and considering a free jet. Most of the work in jet mixing has been done on free jets, and the results (ref. 5) of these experiments and analyses can be used as a basis for estimating eddy viscosity. Prandtl's postulate that eddy viscosity is proportional to the product of half radius and maximum velocity difference has been shown to agree with data in the far jet region, when density is constant and the initial coaxial-stream- to jet-velocity ratio is small. However, there is little agreement on how this postulate should be modified when density varies. Ting and Libby (ref. 6) have extended this formulation to allow for density differences, but adequate experimental verification has not been obtained.

This report describes a computer program that has been developed for isothermal, turbulent jet mixing of two gases. The program is capable of solving problems with large initial velocity and density ratios without the restrictive assumption of constant eddy viscosity. The effects of velocity and density ratios on the mean flow properties are shown; also, the marked influences of the reference density, unspecified in the Ting-Libby formulation, are illustrated.

ANALYSIS

Boundary Layer Equations, Initial and Boundary Conditions

The equations that are used to describe the isothermal, turbulent jet mixing of two gases are the time-averaged continuity equation and boundary layer forms of the momentum and diffusion equations. The von Mises transformation converts the momentum and diffusion equations to forms that satisfy the continuity equation identically. Eddy viscosity is specified empirically by a combination of Prandtl's constant density formulation and a relation proposed by Ting and Libby (ref. 6).

For jet mixing at large Reynolds number, molecular transport is negligible compared with turbulent transport and can be ignored. At low Mach number, density changes result solely from mixing, and for a free jet the pressure is constant. Using capital letters to denote dimensional quantities, the axisymmetric forms of the continuity, momentum, and diffusion equations are as follows (ref. 5):

$$\frac{\partial}{\partial X}(PUR) + \frac{\partial}{\partial R}(PVR) = 0 \tag{1}$$

$$PU\frac{\partial U}{\partial X} + PV\frac{\partial U}{\partial R} = \frac{1}{R}\frac{\partial}{\partial R}\left(PER\frac{\partial U}{\partial R}\right) \tag{2}$$

$$PU\frac{\partial Y}{\partial X} + PV\frac{\partial Y}{\partial R} = \frac{1}{R}\frac{\partial}{\partial R}\left(\frac{PE}{Sc_t} R\frac{\partial Y}{\partial R}\right) \tag{3}$$

(Symbols are defined in appendix A.) The diffusion equation is written in terms of the mass fraction of component 1, the major (or total) constituent of the initial jet. It is assumed that the turbulent momentum and mass fluxes can be represented as the product of an eddy viscosity and the gradient of a time-averaged quantity. In addition, the turbulent Schmidt number Sc_t is used to relate the eddy viscosities for momentum and mass.

The initial and boundary conditions for the problem are as follows:

$$\left.\begin{array}{lll} U = U_j,\ Y = Y_j & 0 \le R < R_j & X = 0 \\ U = U_e,\ Y = Y_e & R > R_j & X = 0 \end{array}\right\} \tag{4}$$

$$\frac{\partial U}{\partial R} = 0,\ V = 0,\ \frac{\partial Y}{\partial R} = 0 \qquad R = 0 \qquad X \ge 0 \tag{5}$$

$$U \rightarrow U_e,\ Y \rightarrow Y_e \qquad R \rightarrow \infty \qquad X \ge 0 \tag{6}$$

These conditions may have to be modified when comparing computer results and experimental data. If the wall thickness of the jet discharge tube is not small compared with the tube diameter, the presence of a wall may significantly influence the early development of the flow. Also, the jet and coaxial-stream velocities will not, in general, be uniform but will have some distribution. If the duct surrounding the coaxial stream is not large compared with the jet discharge tube, the assumption of a coaxial stream of infinite extent is not justified.

The equations can be made dimensionless in terms of the initial jet velocity, mass fraction, density, and radius. When lower-case letters are used to denote dimensionless quantities, the equations become

$$\frac{\partial}{\partial x}(\rho u r) + \frac{\partial}{\partial r}(\rho v r) = 0 \tag{7}$$

$$\rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial}{\partial r}\left(\rho\epsilon r\frac{\partial u}{\partial r}\right) \tag{8}$$

$$\rho u \frac{\partial y}{\partial x} + \rho v \frac{\partial y}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{\rho \epsilon}{Sc_t} r \frac{\partial y}{\partial r} \right) \tag{9}$$

where

$$\rho \epsilon = \frac{PE}{P_j U_j R_j} \tag{10}$$

The dimensionless initial and boundary conditions are as follows:

$$\left. \begin{array}{lll} u = 1, \; y = 1 & 0 \leq r < 1 & x = 0 \\ u = u_e, \; y = y_e & r > 1 & x = 0 \end{array} \right\} \tag{11}$$

$$\frac{\partial u}{\partial r} = 0, \quad v = 0, \quad \frac{\partial y}{\partial r} = 0 \qquad r = 0 \qquad x \geq 0 \tag{12}$$

$$u \rightarrow u_e, \; y \rightarrow y_e \qquad r \rightarrow \infty \qquad x \geq 0 \tag{13}$$

The density and mass fraction of component 1 can be related by the ideal gas law. Thus,

$$\rho = \frac{M}{M_j} = \frac{m_2 - 1 + \frac{1}{Y_j}}{(m_2 - 1)\, y + \frac{1}{Y_j}} \tag{14}$$

where $m_2 = M_2/M_1$. For most applications, the initial jet will be pure component 1, and the initial coaxial stream will be pure component 2. Then, $\rho_e = m_2$ and the mole fraction of component 1 is

$$c = \frac{\rho - \rho_e}{1 - \rho_e} \tag{15}$$

A "compatibility condition" that must be satisfied by the numerical solution can be obtained by evaluating the momentum equation at the centerline; this condition provides a check on mesh size. Thus, when l'Hospital's rule is used to evaluate the indeterminate form that arises,

$$\rho_{\mathbb{C}} u_{\mathbb{C}} \frac{du_{\mathbb{C}}}{dx} = 2\, \rho_{\mathbb{C}} \epsilon_{\mathbb{C}} \left. \frac{\partial^2 u}{\partial r^2} \right|_{\mathbb{C}} \tag{16}$$

von Mises Transformation

The numerical integration can be facilitated by using the von Mises transformation (ref. 7, p. 136) to convert from (r, x) to (ψ, x) coordinates, in which the continuity equation is satisfied identically. Defining a stream function ψ such that

$$\left.\begin{aligned} \frac{\partial \psi}{\partial r} &= \rho u r \\ \frac{\partial \psi}{\partial x} &= -\rho v r \end{aligned}\right\} \tag{17}$$

and using the chain rule for differentiation

$$\left.\frac{\partial}{\partial x}\right|_r = \left.\frac{\partial}{\partial x}\right|_\psi + \left.\frac{\partial}{\partial \psi}\right|_x \left.\frac{\partial \psi}{\partial x}\right|_r = \left.\frac{\partial}{\partial x}\right|_\psi - \rho v r \left.\frac{\partial}{\partial \psi}\right|_x \tag{18}$$

$$\left.\frac{\partial}{\partial r}\right|_x = \left.\frac{\partial}{\partial \psi}\right|_x \left.\frac{\partial \psi}{\partial r}\right|_x = \rho u r \left.\frac{\partial}{\partial \psi}\right|_x \tag{19}$$

results in the following forms of the momentum and diffusion equations:

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial \psi}\left(\rho \epsilon\, \rho u r^2 \frac{\partial u}{\partial \psi}\right) \tag{20}$$

$$\frac{\partial y}{\partial x} = \frac{\partial}{\partial \psi}\left(\frac{\rho \epsilon}{Sc_t}\, \rho u r^2 \frac{\partial y}{\partial \psi}\right) \tag{21}$$

In these coordinates the initial and boundary conditions are as follows:

$$
\left.\begin{array}{llll} u = 1,\ y = 1 & 0 \le \psi < \psi_j & x = 0 \\ u = u_e,\ y = y_e & \psi > \psi_j & x = 0 \end{array}\right\} \tag{22}
$$

$$
\frac{\partial u}{\partial \psi} = 0,\ \frac{\partial y}{\partial \psi} = 0 \qquad \psi = 0 \qquad x \ge 0 \tag{23}
$$

$$
u = u_e,\ y = y_e \qquad \psi \to \infty \qquad x \ge 0 \tag{24}
$$

The transformation obtained by integrating the first of equations (17)

$$
\psi = \int_0^r \rho u r\, dr \tag{25}
$$

can be used to determine ψ.

Eddy Viscosity

It remains to specify the eddy viscosity. Two formulations are required: one for the "near" jet, before the centerline velocity begins to change, where the mixing is more nearly planar; and the other for the "far" jet, where the mixing is truly axisymmetric. No attempt was made to make the eddy viscosity continuous at the point where one formulation replaces the other.

Ting and Libby (ref. 6) have postulated relations between the eddy viscosities in constant-density and variable-density flows. These relations can be written as

$$
\epsilon = \left(\frac{\rho_o}{\rho}\right)^2 \epsilon^* \tag{26}
$$

for the near jet and

$$
\epsilon = \epsilon^* \left(\frac{\rho_o}{\rho}\right)^2 \frac{1}{r^2} \int_0^r 2 \frac{\rho}{\rho_o} r\, dr \tag{27}
$$

for the far jet. The asterisk refers to constant-density flows and ρ_o is a reference density for the flow. Since the reference density is not specified, it must be determined by comparison of calculation and experiment. The value for the centerline eddy viscosity can be obtained by expanding ρ in a Taylor series in r about r = 0, performing the integration, and taking the limit as $r \rightarrow 0$. In this way

$$\epsilon_{\mathbb{C}} = \left(\frac{\rho_o}{\rho_{\mathbb{C}}}\right)\epsilon^*_{\mathbb{C}} \tag{28}$$

The constant-density eddy viscosity can be represented by the following equations (ref. 7) in the near and far jets, respectively,

$$\epsilon^* = k_1 x(u_e - 1) \tag{29}$$

and

$$\epsilon^* = k_2 r_{1/2}(u_e - u_{\mathbb{C}}) \tag{30}$$

For a jet discharging into a quiescent ambient stream, k_2 was found experimentally to be 0.0256 (ref. 7). In terms of these expressions for eddy viscosity, the centerline compatibility conditions for the near and far jets, respectively, become

$$u_{\mathbb{C}} \frac{du_{\mathbb{C}}}{dx} = 2k_1 x\left(\frac{\rho_o}{\rho_{\mathbb{C}}}\right)^2 (u_e - 1) \left.\frac{\partial^2 u}{\partial r^2}\right|_{\mathbb{C}} \tag{31}$$

and

$$u_{\mathbb{C}} \frac{du_{\mathbb{C}}}{dx} = 2k_2 r_{1/2}(u_e - u_{\mathbb{C}})\left(\frac{\rho_o}{\rho_{\mathbb{C}}}\right) \left.\frac{\partial^2 u}{\partial r^2}\right|_{\mathbb{C}} \tag{32}$$

The unspecified reference density is presumably the centerline density, the coaxial-stream density, or a combination of these. It was assumed that a simple linear combination was adequate. The reference density was thus taken to be

$$\rho_o = A\rho_{\mathbb{C}} + B\rho_e \tag{33}$$

where A and B are positive input constants. Restricting the sum of A and B to 1 bounds ρ_o between $\rho_{\mathbb{C}}$ and ρ_e.

Calculations

In addition to the velocity, mass fraction, and mole fraction variations, several other quantities are of interest. The turbulent momentum and mass fluxes are

$$\tau = -\rho\epsilon \frac{\partial u}{\partial r} \tag{34}$$

$$\mu = -\frac{\rho\epsilon}{Sc_t} \frac{\partial y}{\partial r} \tag{35}$$

The width of the jet is characterized by its half radius (i.e., the point at which the local velocity equals the average of the centerline and coaxial-stream velocities).

A gross measure of concentration that can be easily obtained experimentally is the amount of light attenuated by an opaque substance. The attenuation can be related to concentration if the optical properties of the components are known. This technique was used in reference 8 with a bromine jet and a coaxial stream of air. This measure of concentration can be calculated as follows:

$$c* = \int_0^\infty \frac{\rho - \rho_e}{1 - \rho_e} \, dr \tag{36}$$

Gas-Core Nuclear Reactor Calculations

For these calculations, it was assumed that the fuel instantly vaporizes upon entering the reactor. Heat-transfer calculations (ref. 3) were used to estimate average fuel and propellant temperatures so that molecular weights and densities could be calculated.

The major fluid-mechanical figure of merit for the gas-core nuclear reactor is the amount of fuel contained within a given volume

$$W = 2\pi P_j Y_j R_j^3 I \tag{37}$$

where

$$I = \int_0^l \int_0^{d/2} \rho y r \, dr \, dx \tag{38}$$

The calculations were performed for a coaxial stream infinite in extent, whereas in a gaseous nuclear rocket the coaxial stream is, of course, bounded. Indeed, it is now thought that the reactor radius will be only about twice the radius of the jet discharge tube. However, the radial integration in equation (38) was terminated at the position corresponding to a reactor wall in order to estimate the fuel content. Since the radial velocity at this position is not zero in the calculations, the situation to which the calculations apply corresponds to a porous-wall reactor with this distribution of radial velocity at the wall. Alternatively, if the integration is carried out to a constant ψ value corresponding to the initial reactor radius, axial mass flow is constant but the reactor walls are no longer cylindrical. The two additional limitations in applying the results of the calculations to gaseous nuclear reactor geometry are the use of the boundary layer equations close to the jet exit and the absence of an end wall in the calculations. Thus, the results of the calculations are approximations to reactor conditions.

The amount of fuel present in a given volume can be compared with the minimum and maximum amounts that could be present. Fuel content would be a minimum if the jet and the coaxial stream were perfectly mixed before injection into the reactor. Thus,

$$I_{min} = (\rho y)_{av} \int_0^l \int_0^{d/2} r \, dr \, dx = \frac{1}{8} d^2 l \, (\rho y)_{av} \tag{39}$$

If the jet and the coaxial stream are composed of pure component 1 and pure component 2, respectively,

$$Y_{av} = \frac{P_j U_j \pi R_j^2}{P_j U_j \pi R_j^2 + P_e U_e \left(\frac{\pi D^2}{4} - \pi R_j^2 \right)} \tag{40}$$

so that

$$y_{av} = \frac{1}{1 + \rho_e u_e \left(\frac{1}{4} d^2 - 1 \right)} \tag{41}$$

Then,

$$(\rho y)_{av} = \frac{1}{1 + u_e \left(\frac{1}{4} d^2 - 1 \right)} \tag{42}$$

and

$$I_{min} = \frac{1}{8} \frac{d^2 l}{1 + u_e\left(\frac{1}{4} d^2 - 1\right)} \tag{43}$$

The amount of fuel would be a maximum if the initial jet were to remain a cylinder of uniform concentration with the same radius as the jet discharge tube. In this case

$$I_{max} = \int_0^l \int_0^1 r \, dr \, dx = \frac{1}{2} l \tag{44}$$

Numerical Method

In this section, a detailed analysis is presented of the numerical techniques used to solve the equations of coaxial turbulent jet mixing. Figure 2 is a general flow diagram for the numerical solution. The initial difficulty, from a numerical standpoint, is the application of the boundary conditions $u = u_e$ and $y = y_e$ as $\psi \rightarrow \infty$. This difficulty was overcome by defining a parameter ψ_∞, such that as $\psi \rightarrow \psi_\infty$ not only do the functions approach the boundary conditions, but also the derivatives of the functions are restricted to fall below some arbitrarily small parameter. Furthermore, since ψ_∞ can be a numerically large value, a transformation was performed on the independent variable ψ to limit the range of integration from 0 to 1. An implicit finite-difference technique, the Crank-Nicholson method (ref. 9), was employed to solve the system of parabolic equations. Stability is inherent in such a scheme, and a high degree of accuracy can be obtained by a judicious choice of interval size.

Let

$$\overline{\psi} = \frac{\psi}{\psi_\infty} \tag{45}$$

so that

$$\frac{\partial}{\partial \psi} = \frac{1}{\psi_\infty} \frac{\partial}{\partial \overline{\psi}} \tag{46}$$

Applying this linear transformation to equations (20), (21), and the inverse of (25) results in the following equations:

$$\frac{\partial u}{\partial x} = \frac{1}{\psi_\infty^2} \frac{\partial}{\partial \overline{\psi}} \left(\rho\epsilon\ \rho u r^2 \frac{\partial u}{\partial \overline{\psi}} \right) \tag{47}$$

$$\frac{\partial y}{\partial x} = \frac{1}{\psi_\infty^2 Sc_t} \frac{\partial}{\partial \overline{\psi}} \left(\rho\epsilon\ \rho u r^2 \frac{\partial y}{\partial \overline{\psi}} \right) \tag{48}$$

$$r^2 = 2\psi_\infty \int_0^{\overline{\psi}} \frac{d\overline{\psi}}{\rho u} \tag{49}$$

Correspondingly, the initial conditions become

$$\left.\begin{aligned} u &= 1 \qquad 0 \le \overline{\psi} \le \frac{1}{2\psi_\infty} \\ u &= u_e \qquad \overline{\psi} > \frac{1}{2\psi_\infty} \end{aligned}\right\} \tag{50}$$

$$\left.\begin{aligned} y &= 1 \qquad 0 \le \overline{\psi} \le \frac{1}{2\psi_\infty} \\ y &= y_e \qquad \overline{\psi} > \frac{1}{2\psi_\infty} \end{aligned}\right\} \tag{51}$$

The boundary conditions are transformed to

$$\frac{\partial u}{\partial \overline{\psi}} = \frac{\partial y}{\partial \overline{\psi}} = 0 \qquad \overline{\psi} = 0 \tag{52}$$

$$u = u_e,\ y = y_e \qquad \overline{\psi} = 1 \tag{53}$$

Finite-Difference Equations

Consider a linear parabolic equation of the form

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial \bar{\psi}}\left[A(\bar{\psi}, x)\frac{\partial u}{\partial \bar{\psi}}\right] \tag{54}$$

Equations (47) and (48) can be considered of this form if

$$A_u(\bar{\psi}, x) = \frac{\rho\epsilon\,\rho u r^2}{\psi_\infty^2} \tag{55}$$

and

$$A_y(\bar{\psi}, x) = \frac{\rho\epsilon\,\rho u r^2}{\psi_\infty^2 Sc_t} \tag{56}$$

and if the values of the quantities in equations (55) and (56) are initially assumed to be those at the previous axial position. By successively solving equation (54) and recomputing A_u and A_y until no further change occurs, the correct values are obtained.

Now consider a net $R_{i,j}$ constructed on the region of interest. Let the subscript i represent discrete points on the $\bar{\psi}$-coordinate, and j discrete points on the x-coordinate. The points of the $\bar{\psi}$-coordinate are constructed such that $\bar{\psi}_i = (i - 1)\Delta\bar{\psi}$ with i ranging from 1 to N. Thus, the notation $u_{i,j}$ corresponds to the functional value of $u(\bar{\psi}_i, x_j)$. If forward differentiating is used over intervals in the x direction, then equation (54) can be integrated between mesh points to yield

$$\int_{i-\frac{1}{2}}^{i+\frac{1}{2}} \frac{u_{i,j+1} - u_{i,j}}{\Delta x}\, d\bar{\psi} = \left(A\frac{du}{d\bar{\psi}}\right)_{i+\frac{1}{2},\, j+\frac{1}{2}} - \left(A\frac{du}{d\bar{\psi}}\right)_{i-\frac{1}{2},\, j+\frac{1}{2}} \tag{57}$$

If the integrand in equation (57) remains constant over the small interval $\Delta\bar{\psi}$,

$$\frac{\Delta\bar{\psi}}{\Delta x}(u_{i,j+1} - u_{i,j}) = \left(A\frac{du}{d\bar{\psi}}\right)_{i+\frac{1}{2},\, j+\frac{1}{2}} - \left(A\frac{du}{d\bar{\psi}}\right)_{i-\frac{1}{2},\, j+\frac{1}{2}} \tag{58}$$

Central differences are used for the right side of equation (58); values of A and u at half intervals are approximated by the average over the whole interval; and the abbreviations

$$w_i^{\pm} = \frac{A_i + A_{i\pm 1}}{4\Delta\overline{\psi}} \tag{59}$$

are introduced. Substituting these results in equation (58) yields

$$w_i^- u_{i-1,j+1} - \left(w_i^+ + w_i^- + \frac{\Delta\overline{\psi}}{\Delta x}\right) u_{i,j+1} + w_i^+ u_{i+1,j+1}$$
$$= - w_i^- u_{i-1,j} - \left(\frac{\Delta\overline{\psi}}{\Delta x} - w_i^- - w_i^+\right) u_{i,j} - w_i^+ u_{i+1,j} \tag{60}$$

Equation (60) describes a linear set of equations for the unknowns $u_{i,j+1}$. Two additional equations are needed to complete the set in which i ranges from 1 to N. Integrating equation (54) from $i = 1$ to $i = 1\frac{1}{2}$ and using the fact that $(du/d\overline{\psi})_{\overline{\psi}=0} = 0$ gives

$$(u_{1,j+1} - u_{1,j})\left(\frac{\Delta\overline{\psi}}{2\Delta x}\right) = \left(A \frac{du}{d\overline{\psi}}\right)_{1\frac{1}{2}} \tag{61}$$

Following the same procedure used to obtain equation (60), equation (61) can be expressed as follows:

$$-\left(w_1^+ + \frac{\Delta\overline{\psi}}{2\Delta x}\right) u_{i,j+1} + w_1^+ u_{2,j+1} = - w_1^+ u_{2,j} - \left(\frac{\Delta\overline{\psi}}{2\Delta x} - w_1^+\right) u_{i,j} \tag{62}$$

Applying the boundary condition at $\overline{\psi} = 1$ (i.e., i = N) by using equation (60) with i = N and noting that $u_{N+1,j+1} = u_e$ yields

$$w_N^- u_{N-1,j+1} - \left(w_N^+ + w_N^- + \frac{\Delta\overline{\psi}}{\Delta x}\right) u_{N,j+1} = 2w_N^+ u_e - w_N^- u_{N-1,j} - \left(\frac{\Delta\overline{\psi}}{\Delta x} - w_N^+ - w_N^-\right) u_{N,j} \tag{63}$$

The relations given by equations (60), (62), and (63) are written in matrix notation as follows:

$$\underline{B}\,\vec{u}_{j+1} = \vec{d} \tag{64}$$

where the vectors $\vec{u}_{j+1}$ are the unknown quantities. The vector $\vec{d}$ has the following elements:

$$\left.\begin{aligned}
d_1 &= -w_1^+u_{2,j} - \left(\frac{\Delta\overline{\psi}}{2\Delta x} - w_1^+\right)u_{i,j} \\
d_i &= -w_i^-u_{i-1,j} - \left(\frac{\Delta\overline{\psi}}{\Delta x} - w_i^- - w_i^+\right)u_{i,j} - w_i^+u_{i+1,j} \qquad i = 2, 3, \ldots, N-1 \\
d_N &= 2w_N^+u_e - w_N^-u_{N-1,j} - \left(\frac{\Delta\overline{\psi}}{\Delta x} - w_N^+ - w_N^-\right)u_{N,j}
\end{aligned}\right\} \tag{65}$$

The tridiagonal matrix $\underline{B}$, whose superdiagonal and subdiagonal elements are w_i^+ and w_i^-, respectively, has the following diagonal elements:

$$\left.\begin{aligned}
b_{11} &= -\left(w_1^+ + \frac{\Delta\overline{\psi}}{2\Delta x}\right) \\
b_{ii} &= -\left(w_i^+ + w_i^- + \frac{\Delta\overline{\psi}}{\Delta x}\right) \qquad i = 2, 3, \ldots, N-1 \\
b_{NN} &= -\left(w_N^+ + w_N^- + \frac{\Delta\overline{\psi}}{\Delta x}\right)
\end{aligned}\right\} \tag{66}$$

The solution of equation (64) is directly obtained by Gaussian elimination of the subdiagonal elements and a back substitution to obtain $\vec{u}_{j+1}$.

Now that a method to solve equation (54) has been devised, the calculational procedure for solving equations (48) and (49) is as follows:

(1) Initially, at an axial length x_j, use values of r, ρ, u, and y from x_{j-1} to compute A_u and A_y.

(2) Solve for u_{j+1} and y_{j+1}.

(3) Recompute the coefficients A_u and A_y.

(4) Iterate between steps 2 and 3 until the change in A_u and A_y is less than a specified amount.

This procedure was programmed for the IBM 7094 II 7044 Direct Couple System in FORTRAN IV. A listing of the program is given in appendix B. For most cases considered, a value of $\Delta\overline{\psi} = 1/200$ and an initial $\Delta x = 10^{-3}$ gave satisfactory results. However, at larger axial positions, larger values of Δx can be used because of the decaying effects of the initial step profiles. An heuristic approach was used to alter Δx; if the iteration procedure converged in three or less iterations, Δx was increased by 0.5 percent but was limited to a value of 0.4. Running time, of course, varied with initial input; however, an average time of approximately 0.3 minute per unit x could be expected.

Program Input and Output

The input required for a calculation consists of the following information:

(1) The initial ratio of coaxial-stream velocity to jet velocity u_e

(2) The mass fraction of component 1 in the initial jet y_j and in the initial coaxial stream y_e

(3) The ratio of the molecular weight of component 2 to the molecular weight of component 1 m_2

(4) The constants in the eddy viscosity formulation k_1 and k_2

(5) The turbulent Schmidt number Sc_t

(6) The ratio of reactor diameter to jet radius d

(7) The constants in the reference density formulation A and B

(8) The axial positions at which output is desired x

The output listing reproduces the input and, thereafter, at each axial position, gives

(1) Axial position x

(2) Eddy viscosity ϵ

(3) The product of density and eddy viscosity $\rho\epsilon$

(4) The following information for axial velocity, mass fraction, and mole fraction:

(Centerline value - Coaxial-stream value)/(1 - Coaxial-stream value)

The radial variations with stream function, also converted to radial position r, and the ratio of radial position to half radius $r/r_{1/2}$ for axial velocity, mass fraction, and mole fraction are provided in the following form:

$$\frac{(\text{Local value - Ambient stream value})}{(\text{Centerline value - Ambient stream value})}$$

The following quantities are also listed:

(1) Momentum flux normalized with the square of the centerline velocity $\tau/u_{\mathfrak{C}}^2$

(2) Mass flux normalized with the product of centerline velocity and mass fraction $\mu/u_{\mathfrak{C}}y_{\mathfrak{C}}$

(3) Eddy viscosity and the product of density and eddy viscosity divided by their centerline values $\epsilon/\epsilon_{\mathfrak{C}}$ and $\rho\epsilon/(\rho\epsilon)_{\mathfrak{C}}$

Both sides of the centerline compatibility condition are printed next, followed by the values of velocity and density at the largest stream function position in the calculation. Finally, the "line of sight" concentration c^*, the dimensionless mass of component 1 I, and the ratio of mass of component 1 to initial mass (ψ ratio) are listed.

RESULTS AND DISCUSSION

Sample results from the computer program are presented to illustrate the mixing of a heavy, slow-moving jet and a lighter, faster-moving coaxial stream. First, however, a limiting case is discussed.

Schlichting (ref. 7, p. 607) presents a similarity solution for an isothermal, turbulent free jet mixing with a quiescent ambient stream of the same composition that is valid far downstream. Schlichting's value for the proportionality constant k_2 in the far-jet eddy viscosity formulation was adopted in order that the numerical solution represent this limiting case. The value of the constant of proportionality k_1 in the near-jet eddy viscosity formulation was chosen so that the numerical solution agreed with the similarity solution far downstream.

Figure 3(a) shows a comparison of centerline velocity and half radius calculated from the similarity solution and the results of the numerical solution using two different values of k_1. A value of $k_1 = 0.75\times10^{-3}$ leads to good agreement downstream and was therefore used in subsequent calculations. Radial velocity profiles rapidly change from the initial step profile and gradually merge into the similarity profile. Figure 3(b) illustrates that at an axial position of 16 jet radii the profile is almost similar, whereas at 50 jet radii the agreement is essentially exact. In these and subsequent calculations, the near jet formulation for eddy viscosity was used until $(u_{\mathrm{c}\!\!\!\mathrm{L}} - u_e)/(1 - u_e) = 0.99$; thereafter the far jet formulation was used. The calculation was repeated for 0.98 in order to see whether the particular choice of 0.99 was critical. Although there was some difference initially, the difference between the two solutions was soon indistinguishable.

Choosing the proportionality constants in the eddy viscosity formulations by the method just discussed leads to a discontinuity in eddy viscosity at the axial position where one formulation replaces the other. It was felt that agreement with the similarity solution was more important than a continuous variation of eddy viscosity. Figures 4 and 5 show typical calculations of the axial and radial variations of eddy viscosity and the product of density and eddy viscosity, respectively. In both cases, the radial variation is much greater in the near jet than farther downstream. The product of density and eddy viscosity varies less in the radial direction than does eddy viscosity.

The effects of two different initial velocity ratios on velocity and mass fraction profiles are illustrated in figures 6 and 7. The higher velocity ratio, of course, results in a more rapid decay in centerline values and a narrower jet. In all calculations, the generally accepted value of 0.7 (ref. 5, p. 422) was used for the turbulent Schmidt number.

The effects of two different initial density ratios on velocity and mass fraction profiles are illustrated in figures 8 and 9. As expected, the lighter jet decays faster and results in a narrower jet.

The influence of reference density on velocity and mass fraction profiles are shown in figures 10 and 11. The formulation based on centerline density leads to much more rapid decay of centerline values and to a narrower jet than does the formulation based on ambient stream density. The formulation based equally on centerline and ambient stream densities falls between these results. These figures demonstrate that it will be necessary to determine experimentally if any of these formulations are adequate.

Figures 12 and 13 show the effect of initial velocity ratio and initial density ratio on the mass of the major jet component contained within a given volume. The maximum and minimum values are included for comparison.

CONCLUDING REMARKS

A computer program to describe isothermal, turbulent jet mixing of two gases was written using the axisymmetric forms of the boundary layer momentum and diffusion equations. The coaxial stream is considered to be infinite in extent. Eddy viscosity is represented by an expression that provides for both radial and axial variation. Typical running time is less than 1 minute to produce time-averaged velocity and mass fraction distributions.

Experimental data are required for further progress. Constant-density experiments at large initial velocity ratios will determine if the numerical value of k_2 used is appropriate. If not, the large initial velocity ratio data can be used to determine a new value. The value of k_1 can also be obtained from the same experiments. With the values of k_1 and k_2 determined, variable-density experiments at large velocity ratios can be used to determine a suitable reference density in the eddy viscosity formulations.

In the comparison of computer results and experimental jet-mixing data, the initial conditions on the equations may have to be modified to account for the finite wall thickness of the jet discharge tube and for the distribution of velocities in the initial jet and the coaxial stream. For gas-core nuclear reactor calculations, the absence of an end-wall boundary condition is probably a serious restriction. In addition, the assumption of constant pressure and the use of the boundary layer equations near the jet exit are approximations. These restrictions can be removed by using the full Navier-Stokes equations rather than the boundary layer equations. However, the problem of turbulence and a method to characterize an eddy viscosity remain.

For unbounded turbulent jet mixing, the computer program provides a rapid solution that reduces to the similarity solution when the density is constant and the coaxial stream is quiescent. Improved values for the proportionality constants in the eddy viscosity formulations, or an entirely new expression, can easily be incorporated.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, September 8, 1967,
122-28-02-16-22.

APPENDIX A

SYMBOLS

A, B	constants in reference density formula
A_u, A_y	quantities defined by eqs. (55) and (56)
$\underline{B}$	coefficient matrix
b	elements of $\underline{B}$ matrix
C	mole fraction of component 1
c	dimensionless mole fraction of component 1, C/C_j
c^*	line of sight concentration
D	reactor diameter
d	dimensionless reactor diameter, D/R_j; elements of $\vec{d}$ vector
$\vec{d}$	vector defined by eqs. (65)
I	dimensionless mass of component 1, $\int_0^l \int_0^{d/2} \rho yr \, dr \, dx$
$k_1 k_2$	constants in eddy viscosity formulations
L	reactor length
l	dimensionless reactor length, L/R_j
M	molecular weight
m	dimensionless molecular weight, M/M_1
N	upper limit of i
R	radial position; finite difference net
r	dimensionless radial position, R/R_j
$r_{1/2}$	dimensionless half radius (i.e., position at which $(u - u_e)/(1 - u_e) = 1/2$)
Sc_t	turbulent Schmidt number
U	axial velocity
u	dimensionless axial velocity, U/U_j
$\vec{u}$	velocity vector
V	radial velocity

v	dimensionless radial velocity, V/U_j
W	amount of fuel contained within a given volume, $2\pi P_j Y_j R_j^3 I$
w	abbreviation defined by eq. (59)
X	axial position
x	dimensionless axial position, X/R_j
Y	mass fraction of component 1
y	dimensionless mass fraction of component 1, Y/Y_j
E	eddy viscosity
ϵ	dimensionless eddy viscosity, $E/U_j R_j$
ϵ^*	dimensionless constant-density eddy viscosity
P	density
ρ	dimensionless density, P/P_j
ψ	stream function
$\overline{\Psi}$	normalized stream function, ψ/ψ_∞
ψ_∞	maximum value of ψ

Subscripts:

av	average
℄	centerline
e	coaxial stream
i	point on $\overline{\Psi}$ coordinate
j	jet; point on x coordinate for numerical solution
max	maximum
min	minimum
o	reference
1	major component of jet
2	major component of coaxial stream

APPENDIX B

PROGRAM LISTING

```
$ID     YED1581 C A TODD COAXIAL FLOW
C              A COMPUTER PROGRAM FOR CALCULATING ISOTHERMAL
C                 TURBULENT JET MIXING OF TWO GASES
C
C     SYMBOL                     DEFINITION
C
C     AP      THE A COEFF. FOR THE DIFFUSION EQN.
C     AU      THE A COEFF. FOR THE MOMENTUM EQN.
C     BP      THE B COEFF. FOR THE DIFFUSION EQN
C     BU      THE B COEFF. FOR THE MOMENTUM EQN.
C     DPSI    THE INTERVAL SIZE IN THE PSI DIRECTION
C     DX      THE INTERVAL SIZE IN THE X DIRECTION
C     EA      COEFF. A, FOR REFERENCE DENSITY CALCULATION
C     EB      COEFF. B, FOR REFERENCE DENSITY CALCULATION
C     EPS     RHO TIMES THE EDDY VISCOSITY
C     I       INDEX VARIABLE
C     IHALF   VALUE OF I WHEN R=R-1/2
C     ITER    THE ITERATION COUNTER
C     J       INDEX VARIABLE
C     K       A COUNTER TO CONTROL THE OUTPUT
C     KGO     A LOGICAL VARIABLE TO CONTROL THE CALCULATION OF THE
C             CONTAINMENT FACTOR
C     NCOPY   THE NUMBERS OF COPIES WANTED-- NORMALLY 1
C     NPTS    THE NUMBER OF POINTS IN THE PSI DIRECTION
C     NPTSX   THE NUMBER OF X VALUES TO BE OUTPUTED
C     NORMPO  THE EUCLIDIAN NORM OF RHO AT X-DX
C     NORMP2  THE EUCLIDIAN NORM OF RHO AT X
C     NORMUO  THE EUCLIDIAN NORM OF U AT X-DX
C     NORMU2  THE EUCLIDIAN NORM OF U AT X
C     PMAX    PSI-INFINITY AT X
C     PMAXOD  PSI-INFINITY AT X-DX
C     PSI     THE INDEPENDENT VARIABLE
C     PSIMO   PSI-1/2 AT X-DX
C     PSIM1   PSI-1/2 AT X
C     PTS     THE NUMBER OF INTERVALS IN THE PSI DIRECTION
C     PTSX    THE NUMBER OF OUTPUT X'S
C     PZERO   THE PSI-RATIO
C     RHALFO  R-1/2 AT X-DX
C     RHOE    THE EDGE DENSITY
C     RHOR    REFERENCE DENSITY
C     RHOO    VALUE OF RHO AT X-DX
C     RHO1    A GUESSED VALUE OF RHO AT X
C     RHO2    A COMPUTED VALUE OF RHO AT X
C     RO      VALUE OF R AT X-DX
C     R1      A GUESSED VALUE OF R AT X
C     R2      A COMPUTED VALUE OF R AT X
C     SC      THE TURBULENT SCHMIDT NUMBER
C     SD      A DIAMETER
C     SM2     A MOLECULAR WEIGHT RATIO
C     SUM     TEMPORARY STORAGE
C     SUMI    TEMPORARY STORAGE
C     TOL     THE TOLERANCE TO TERMINATE THE ITERATION
C     UE      THE EDGE VELOCITY
C     UO      U AT X-DX
C     UM      INTERPOLATED U AT X-DX
C     U1      A GUESSED VALUE OF U AT X
C     U2      A COMPUTED VALUE OF U AT X
C     XI      THE CONTAINMENT FACTOR
C     XO      THE PREVIOUS VALUE OF THE INDEPENDENT VARIABLE, X
C     X1      THE CURRENT VALUE OF X
```

```
C     XK1      A TURBULENCE FACTOR FOR EDDY VISCOSITY IN THE NEAR JET
C     XK2      A TURBULENCE FACTOR FOR EDDY VISCOSITY IN THE FAR JET
C     XM       A CONTROL VARIABLE
C              XM=0. NORMAL INPUT
C              XM=1. NORMAL INPUT PLUS RESTART DUMP CARDS
C     XPCH     VALUES OF X TO BE OUTPUTED
C     YE       THE EDGE VALUE OF Y
C     YJ       THE JET VALUE OF Y
C     YM       INTERPOLATED Y AT X-DX
C     YO       VALUE OF Y AT X-DX
C     Y1       GUESSED Y AT X
C     Y2       COMPUTED Y AT X
C
C                               INPUT
C                          CARD 1----FORMAT 8F10.X
C        UE                YJ       YE       SM2        XK1        XK2        SC        TOL
C                          CARD 2----FORMAT 8F10.X
C        PTS      PTSX       PMAX            SD         EA         EB         XM      NCOPY
C                          CARD 3+---FORMAT 8F10.X
C   XPCH(1)   XPCH(2)    ---             ETC        ---                             XPCH(8)
C
      COMMON/DATUM/ RO,UO,RHOO,PSI,       NPTS,UE,YJ,YE,SM2,XK1,
     1XK2,SC,RHOE,DPSI,YO,RHO2,U2,R2,PSIM1,RHOE1,RHALF1,X1,IHALF,Y2
      COMMON RHOR,EPS(500)
      DIMENSION UO(500),U1(500),U2(500),RHOO(500),RHO1(500),RHO2(500),
     1RO(500),R1(500),R2(500),V1(500),V(500),BU(500),
     2BP(500),XPCH(50),PSIPCH(50)     ,PSI(500)
      DIMENSION YO(500),Y1(500),Y2(500)
      DIMENSION YM(500),UM(500)
      REAL NORMUO,NORMU1,NORMU2,NORMPO,NORMP1,NORMP2
      SUB(X1,Y1,X2,Y2,X)=(X-X1)/(X2-X1)*(Y2-Y1)+Y1
       COMMON XO(1),PMAX ,NORMUO,NORMPO,DX,XZ,RHALFO,XI,K   ,PZERO
C
C     READ ALL REQUIRED INPUT DATA AND INITIALIZE VALUES OF
C                               STARTING DX.
C
C
    1 READ(5,400)UE,YJ,YE,SM2,XK1,XK2,SC,TOL,PTS,PTSX   ,PMAX ,SD ,EA,EB
     1,XM,TIB
      NCOPY=TIB
      M=XM
      PMAXOD=PMAX
      NPTS=PTS+1.
      NPTSX=PTSX
      T=SM2-1.
      RHOE=(YJ*T+1.)/(YE*T+1.)
      DPSI=1./PTS
      K=1
      READ(5,400)(XPCH(I),I=1,NPTSX)
      XI=0.
      DO 32 I=1,NCOPY
   32 CALL INITAL(M)
      KGO=1
      IF(SM2.GT.1.) KGO=2
      WRITE(6,471) SD  ,EA,EB
      IF(M.NE.0) GO TO 100
      DO 9969 I=1,NPTS
      J=I
 9969 IF(RO(I).GE.SD/2.) GO TO 9970
 9970 PZERO=PSI(J)
```

```
      IF(J.EQ.NPTS) PZERO=PZERO+RHOE*UE/2.*(SD**2/4.-RO(NPTS)**2)
     1/PMAX
  471 FORMAT(2HKD,G13.5,2X,1HA,G13.5,2X,1HB,G13.5 )
  400 FORMAT(8F10.7)
  100 NORMP1=NORMPO
      NORMU1=NORMUO
      PMAX=PMAXOD
C
C    SET FIRST GUESS OF(X+DX)=U(X),Y(X+DX)=Y(X),AND R(X+DX)=R(X)
C
      DO 10 I=1,NPTS
      RHO1(I)=RHOO(I)
      U1(I)=UO(I)
      Y1(I)=YO(I)
   10 R1(I)=RO(I)
      X1=XO+DX
       ITER=0
      NPTS1=NPTS
C
C     IF PSI-MAX HAS CHANGED INTERPOLATE VECTORS TO CORRESPOND
C                               TO NEW LENGTH.
C
  101 DO 102 I=1,NPTS
      YM(I)=YO(I)
  102 UM(I)=UO(I)
      IF(PMAX.EQ.PMAXOD) GO TO 50
      DO 103 I=1,NPTS
      V1(I)=PSI(I)*PMAX
  103 V(I)=PSI(I)*PMAXOD
      DO 104 I=1,NPTS
      CALL SINTP(V,UO,NPTS,V1(I),UM(I))
  104 CALL SINTP(V,YO,NPTS,V1(I),YM(I))
C                COMPUTE PSI-MAX AND PSI-1/2 AND RHO-EPSILON
C
   50 TEST=(U1(1)-UE)/(1.-UE)
      RHOR=EA*RHO1(1)+EB*RHOE
      IF(TEST.LE..99)GO TO 12
      RHOE1=XKI*X1*ABS(UE-1.)
      IHALF=0
      DO 90 I=1,NPTS
   90 EPS(I)=(RHOR/RHO1(I))**2*RHOE1
      GO TO 61
   12 TEST=.5*(UE+U1(1))
      IF(UE.GT.1.) GO TO 490
      DO 14 I=1,NPTS
      IHALF=I
      IF(U1(I).GE.TEST) GO TO 14
      GO TO 15
   14 CONTINUE
  490 DO 491 I=1,NPTS
      IHALF=I
      IF(U1(I).LE.TEST) GO TO 491
      GO TO 15
  491 CONTINUE
C
C
C     COMPUTE VALUES OF AU,AP,BU,AND BP AND SOLVE FOR NEXT
C                      APPROXIMATION OF U,Y,RHO,AND R.
C
   15 DO 456 I=1,IHALF
```

```
  456 V(I)=PMAX/U1(I)
      CALL FNTGRL(IHALF,DPSI,V,V1)
      RHALF1=SQRT(2./RHOR    *V1(IHALF))
      RHOE1=XK2*RHALF1          *ABS(UE-U1(1))
   13 DO 17 I=1,NPTS
   17 V(I)=PMAX/U1(I)
      CALL FNTGRL(NPTS,DPSI,V,V1)
      EPS(1)=RHOE1*RHOR/RHO1(1)
      DO 62 I=2,NPTS
   62 EPS(I)=2.*RHOR*RHOE1/RHO1(I)**2/R1(I)**2*V1(I)
   61 DO 455 I=1,NPTS
       AU=1.
       AP=1.
       BU(I)=EPS(I)*RHO1(I)**2*U1(I)*R1(I)**2
      BU(I)=BU(I)/PMAX**2
  455 BP(I)=BU(I)/SC
      CALL SOLVE(AU,BU,NPTS,UE,U2,NORMU2,DX,DPSI,UM)
      CALL SOLVE(AP,BP,NPTS,YE,Y2,NORMP2,DX,DPSI,YM)
      T=SM2-1.
      T1=1./YJ
      DO 7777 I=1,NPTS
 7777 RHO2(I)=(T+T1)/(Y2(I)*T +T1)
       NPTS2=NPTS1
       DO 18 I=1,NPTS2
   18 V(I)=PMAX/RHO2(I)/U2(I)
      CALL FNTGRL(NPTS2,DPSI,V,V1)
      DO 19 I=1,NPTS2
   19 R2(I)=SQRT(2.       *V1(I))
      DO 11 I=1,NPTS1
   11 V(I)=(U1(I)-UE)/(1.-UE)
      CALL FNTGRL(NPTS1,DPSI,V,V1)
      DO 7070 I=1,NPTS
      I1=I
      IF(V1(I).GT..495*PMAX) GO TO 7071
 7070 CONTINUE
 7071 PSIM1=PSI(I1)
      SUMI=0.
      DO 460 I=2,NPTS
      GO TO (461,462) ,KGO
  461 SUM=Y2(I-1)
      GO TO 463
  462 SUM=1.-Y2(I-1)
  463 IF(R2(I).GT.SD/2.) GO TO 460
      SUMI=SUMI+SUM*RHO2(I-1)*R2(I-1)*(R2(I)-R2(I-1))
  460 CONTINUE
C
C          CHECK TO SEE IF CONVERGENCE CRITERIA HAS BEEN MET.
C
      IF(ABS((NORMU2-NORMU1)/NORMU2).GT.TOL) GO TO 20
      IF(ABS((NORMP2-NORMP1)/NORMP2).GT.TOL) GO TO 20
      TEST=(U2(NPTS)-U2(NPTS-1))/DPSI
      IF(TEST.GT..001*PMAX) GO TO 70
      DEBUG X1,DX,PMAX
      DEBUG (PSI(I),I=1,NPTS,20)
      DEBUG (R2 (I),I=1,NPTS,20)
      DEBUG (U2 (I),I=1,NPTS,20)
C
C                 CONVERGENCE CRITERIA HAS BEEN MET
C                 CHECK WHETHER DX CAN BE INCREASED
C
```

```
      CALL TIMLFT(TIM1)
C
C    IF THE TIME REMAINING IS LESS THAN .1 MIN, DUMP FOR RESTART.
C
      IF(TIM1/3600..GT..1) GO TO 3333
      XZ=K
      CALL BCDUMP(XO(1),XO(8))
      CALL BCDUMP(PSI(1),PSI(NPTS))
      CALL BCDUMP(RO(1),RO(NPTS))
      CALL BCDUMP(UO(1),UO(NPTS))
      CALL BCDUMP(RHOO(1),RHOO(NPTS))
      CALL BCDUMP(YO(1),YO(NPTS))
      STOP
C
C     IF ITER IS LESS THAN 3, INCREASE DX.
C
 3333 IF(ITER.LT.3) DX=1.05*DX
      IF(DX.GT..4) DX=.4
      XI=XI+DX*SUMI
C              SHOULD WE PUNCH OUT AT THIS X
C
      IF(X1.GT.XPCH(K)) GO TO 30
C
   60 XO=X1
      PMAXOD=PMAX
      NORMUO=NORMU2
      NORMPO=NORMP2
      RHALFO=RHALF1
      PSIMO=PSIM1
      RHOEO=RHOE1
      DO 21 I=1,NPTS
      UO(I)=U2(I)
      RHOO(I)=RHO2(I)
      YO(I)=Y2(I)
   21 RO(I)=R2(I)
      GO TO 100
C
C              NO CONVERGENCE
C
   20 NORMU1=NORMU2
      NORMP1=NORMP2
      ITER=ITER+1
      DO 22 I=1,NPTS
      U1(I)=U2(I)
      RHO1(I)=RHO2(I)
      Y1(I)=Y2(I)
   22 R1(I)=R2(I)
      GO TO 50
C
C              PUNCH OUTPUT
C
   30 DO 31 I=1,NCOPY
   31 CALL OUTPUT(XPCH(K))
      WRITE(6,470)XI
      DO 9971 I=1,NPTS
      J=I
 9971 IF(R2(I).GE.SD/2.) GO TO 9972
 9972 TEST=PSI(J)
      IF(J.EQ.NPTS)TEST=TEST+RHOE*UE/2.*(SD**2/4.-R2(NPTS)**2)
     1/PMAX
```

```
      RATP=TEST/PZERO
      WRITE(6,479) RATP
      WRITE(6,482) SUMI
  482 FORMAT(5H I(X),2X,G13.5 )
  479 FORMAT(10HKPSI-RATIO,2X,G11.4)
  470 FORMAT(2HKI,G13.5)
      K=K+1
      IF(X1       .LT.XPCH(K))GO TO 60
      GO TO 1
C           MUST CHANGE PSI-INF
C
   70 PMAX=PMAX+2.
      GO TO 101
      END
$IBFTC SOLVE
C
C   A ROUTINE TO SOLVE A PARABOLIC EQUATION BY THE CRANK-NICHOLSONS
C                                    ALGORITHM.
C
      SUBROUTINE SOLVE(A,B,N,HMAX,H,NORMH,DX,DPSI,HO)
      REAL NORMH
      DIMENSION        B(500),H(500),SB(500),SD(500),HO(500),SA(500),
     1SC(500),WP(500),WM(500)
      N1=N-1
      SA(1)=0.
      T=A/4./DPSI
      T1=DPSI/DX
       DO 5 I=2,N
    5 WM(I)=T*(B(I)+B(I-1))
      DO 6 I=1,N1
    6 WP(I)=T*(B(I+1)+B(I))
      SB(1)=-(WP(1)+T1/2.)
      SC(1)=WP(1)
      SD(1)=(WP(1)-T1/2.)*HO(1)-WP(1)*HO(2)
      N=N-1
      SA(N)=WM(N)
      SB(N)=-(WP(N)+WM(N)+T1)
      SD(N)=-1.*WP(N)*HMAX-WM(N)*HO(N-1)-(-WM(N)+T1-WP(N))*HO(N)
     1-WP(N)*HO(N+1)
       N1=N-1
       DO 7 I=2,N1
      SA(I)=WM(I)
      SC(I)=WP(I)
      SB(I)=-(WM(I)+WP(I)+T1)
    7 SD(I)=-WM(I)*HO(I-1)-WP(I)*HO(I+1)-(T1-WM(I)-WP(I))*HO(I)
      DO 2 I=2,N
      SB(I)=SB(I)-SA(I)/SB(I-1)    *SC(I-1)
    2 SD(I)=SD(I)-SA(I)*SD(I-1)/SB(I-1)
      H(N)=SD(N)/SB(N)
      NORMH=H(N)**2
      DO 3 I=1,N1
      J=N-I
      H(J)=(SD(J)-SC(J)*H(J+1))/SB(J)
    3 NORMH=NORMH+H(J)**2
      N=N+1
      H(N)=HMAX
      NORMH=SQRT(NORMH)
      RETURN
      END
$IBFTC OUTPUT
```

```
      SUBROUTINE OUTPUT(XX)
      COMMON/DATUM/ RO,UO,PO,PSI,N1,UE,YJ,YE,SM2,XK1,XK2,SC,RHOE,DPSI,
     1YO,P2,U2,R2,PM1,PE1,RH2,X1,IH,Y2
      COMMON RHOR,EPS(500)
      DIMENSION P2(500),U2(500),R2(500),PO(500),UO(500),RO(500),
     1PSI(500),PSIP(500),PX(500),UX(500),RX(500),ROX(500),URAT(500),
     2YRAT(500),POP(500),TAU(500),XMU(500) ,YO(500),YX(500),Y2(500)
      DIMENSION ROX1(500),EP(500),RHOEP(500)
      COMMON Z1,PMO
      SUB(X1,Y1,X2,Y2,X)=(X-X1)/(X2-X1)*(Y2-Y1)+Y1
      PMA=SUB(XO,PMO,X1,PM1,XX)
       DO 91  I=1,N1
   91  PSIP(I)=PSI(I)*PMO
      DPSI=PSIP(2)-PSIP(1)
      DO 51  I=1,N1
      IH=I
      T=(U2(I)-UE)/(U2(1)-UE)
      IF(T.GT..5) GO TO 51
       GO TO 52
   51 CONTINUE
   52 RH1=R2(IH)
   50 PMX=1.
      SQT=1.
   11 RHOEX=PE1   /SQT
      RHOX=SUB(XO,PO(1),X1,P2(1),XX)
      IF(RHOE.NE.1.)
     1RHOX=(RHOX-RHOE)/(1.-RHOE)
      UOX=SUB(XO,UO(1),X1,U2(1),XX)
      URATX=(UOX-UE)/(1.-UE)
      T=0.
      IF(SM2.NE.1.) T=1./YJ/(SM2-1.)
                   YOX=SUB(XO,YO(1),X1,Y2(1),XX)
      YRATX=0.
       IF(YE.NE.1.) YRATX=(YOX-YE)/(1.-YE)
C
      W=SM2-1.
      W1=1./YJ
      DO 1 I=1,N1
      YX(I)=SUB(XO,YO(I),X1,Y2(I),XX)
      PX(I)=(W+W1)/(YX(I)*W+W1)
      UX(I)=SUB(XO,UO(I),X1,U2(I),XX)
      RX(I)=SUB(XO,RO(I),X1,R2(I),XX)
      ROX(I)=0.
      IF(IH.NE.0)
     1ROX(I)=RX(I)/RH1
      URAT(I)=(UX(I)-UE)/(UOX-UE)
      XMU(I)=YX(I)*PX(I)*UX(I)**2
      POP(I)=0.
      IF(RHOE.NE.1.)
     1 POP(I)=(PX(I)-RHOE)/(PX(1)-RHOE)
      YRAT(I)=0.
      IF(YE.NE.YJ) YRAT(I)=(YX(I)-YE)/(YOX-YE)
      ROX1(I)=RX(I)/XX
    1 CONTINUE
      DO 521 I=1,N1
      EP(I)=EPS(I)/EPS(1)
  521 RHOEP(I)=PX(I)*EPS(I)/PX(1)/EPS(1)
      TAU(1)=PMX*(UX(2)-UX(1))/DPSI
      TAU(N1)=PMX*(UX(N1)-UX(N1-1))/DPSI
      XMU(1)=(YX(2)-YX(1))/DPSI
```

```
      XMU(N1)=(YX(N1)-YX(N1-1))/DPSI
      N3=N1-1
      DO 2 I=2,N3
      TAU(I)=PMX*(UX(I+1)-UX(I-1))/2./DPSI
    2 XMU(I)=(YX(I+1)-YX(I-1))/2./DPSI
      DO 3 I=1,N1
      T=-RHOEX*PX(I)*UX(I)*RX(I)    /SQT
      TAU(I)=TAU(I)*T   /UOX**2
    3 XMU(I)=XMU(I)*T/SC     /YOX/UOX
      RHOEPX=PX(1)*EPS(1)
      RHX=RH1
      WRITE(6,500) XX,URATX,YRATX,RHOX,EPS(1),RHOEPX   ,RHX
       IMOD=10
      K=1
      TEST=PSIHF
      ISTRT=1
   32 DO 6 I=ISTRT,N1
      N=I
      IF(PSIP(I).GT.TEST) GO TO 7
    6 CONTINUE
    7 MOD=N/IMOD
      IF(MOD.LT.1) MOD=1
      IF(K.EQ.2) N=N+2
      DO 4 I=ISTRT,N,MOD
      RXXX=RX(I)/SQT
      IF(SM2.EQ.1.)XMU(I)=0.
      T5=YX(I)*PX(I)
    4 WRITE(6,501) PSIP(I),RXXX,ROX(I),URAT(I),YRAT(I),POP(I),TAU(I)
     1,XMU(I),EP(I),RHOEP(I)        ,T5
      GO TO (30,31),K
   30 ISTRT=N+1
      TEST=PMA
      IMOD=10
      K=2
      GO TO 32
   31 D1=     UOX*(U2(1)-UO(1))/(X1-XO)    *PX(1)
      D2=4./RX(2)**2*RHOEX*(UX(2)-UX(1))     *RHOR
      T=(UOX-UE)/(1.-UE)
      IF(T.GT..99) D1=D1*PX(1)
      IF(T.GT..99) D2=D2*RHOR
      WRITE(6,600) D1,D2
  600 FORMAT(32HKCENTERLINE COMPATIBILITY VALUES  2G15.5 )
      WRITE(6,601) UX(N1),PX(N1)
  601 FORMAT(7H UMAX= G13.5,3X,9HRHO-MAX= G13.5   )
       N3=N1-1
      CSTAR=0.
      IF(RHOE.EQ.1.) GO TO 602
      DO 66 I=1,N3
      DR=RX(I+1)-RX(I)
   66 CSTAR=CSTAR+(PX(I)-RHOE)/(1.-RHOE)*DR
  602 CONTINUE
      WRITE(6,520) CSTAR
  520 FORMAT(3HKC*,G13.5)
      K=1
      RETURN
  500 FORMAT(19H1    AXIAL-LENGTH,X,G11.4,5X,14H(UO-UE)/(1-UE),G11.4,
     15X,14H(YO-YE)/(1-YE),G11.4,14X,14H(PO-PE)/(1-PE),G11.4 /
     213X,6HEPS-O ,G11.4,12X,7HRHOEPSO,G11.4,14X,5HR-1/2,G11.4,12X,
     3//
     4 23H U-RATIO=(U-UE)/(UO-UE)/  23H Y-RATIO=(Y-YE)/(YO-YE)      /
```

```
     523H P-RATIO=(P-PE)/(PO-PE) /
     6,26H TAU NORMALIZED BY 1/UO**2 / 25H MU NORMALIZED BY 1/UO/YO  //
     54H PSI,7X,1HR,10X,9HR/(R-1/2),2X,7HU-RATIO,4X,7HY-RATIO,4X,
     69HP-RATIO  ,2X,3HTAU,8X,2HMU,9X,
     7 12HEPS/(EPS-O ) ,1X,17HRHOEPS/(RHOEPS-O)    ,
     22X,5HRHO*Y  )
  501 FORMAT(10G11.4,6X,G11.4)
      END
$IBFTC INITAL
      SUBROUTINE INITAL(M)
      COMMON/DATUM/ R,U,RHO,PSI,     N,UE,YJ,YE,SM2,XK1,XK2,SC,RHOE,
     1DPSI,YO
      DIMENSION R(500),U(500),RHO(500),PSI(500),V(500),V1(500)  ,YO(500)
      REAL NU,NP
      COMMON RHOR,EPS(500)
      COMMON X(1),PMO,NU,NP,DX,XK,RHFO,XI,K   ,PZERO
      IF(M.EQ.1) GO TO 100
C
       T=(SM2-1.)*YJ
      PSI(1)=0.
      NU=0.
       NP=0.
      DO 1 I=2,N
    1 PSI(I)=PSI(I-1)+DPSI
      DO 8 I=1,N
      IF(PSI(I).LE..5/PMO) GO TO 4
       U(I)=UE
      RHO(I)=RHOE
      GO TO 3
    4 U(I)=1.
      RHO(I)=1.
    3 NU=NU+U(I)**2
      YO(I)=1.
      IF(T.NE.0.) YO(I)=(YJ/RHO(I)*(1.+1./T)-YJ/T)/YJ
      NP=NP+YO(I)**2
    8 CONTINUE
      NU=SQRT(NU)
      NP=SQRT(NP)
   20 CONTINUE
   51 DO 5 I=1,N
    5 V(I)=2./RHO(I)/U(I)    *PMO
      CALL FNTGRL(N,DPSI,V,V1)
      DO 7 I=1,N
    7 R(I)=SQRT(V1(I))
      RHFO=0.
      IF(M.EQ.2) RETURN
      X=0
      DX=1.E-2
      GO TO 101
  100 CALL BCREAD(X(1),X(9))
      CALL BCREAD(PSI(1),PSI(N))
      CALL BCREAD(R(1),R(N))
      CALL BCREAD(U(1),U(N))
      CALL BCREAD(RHO(1),RHO(N))
      CALL BCREAD(YO(1),YO(N))
      PMO=X(2)
      NU=X(3)
      NP=X(4)
      DX=X(5)
      K=X(6)
```

```
      RHFO=X(7)
      XI=X(8)
      PZERO=X(9)
      DEBUG (X(I),I=1,8)
      DEBUG (YO(I),I=1,N)
  101 WRITE (6,500)UE,YJ,YE,SM2,XK1,XK2,SC        ,X
      SQT=SQRT(PMO)
      MOD=N/20
      DO 6 I=1,N,MOD
      PSIP=PSI(I)     *PMO
      RP=R(I)
    6 WRITE(6,501)PSIP,RP,U(I),RHO(I)
      RETURN
  500 FORMAT(1H1,30X,30HINPUT FOR TURBULENT JET MIXING //
     17X,2HUE,G11.4,7X,2HYJ,G11.4,7X,2HYE,G11.4,7X,2HM2,G11.4,7X,2HK1,
     2G11.4,7X,2HK2,G11.4/7X,2HSC,G11.4 //
     330X,24HINITIAL PROFILES FOR X= G11.4   //
     54H PSI,9X,1HR,12X,1HU,12X,3HRHO )
  501 FORMAT(4G13.5)
      END
$IBFTC SINTP
      SUBROUTINE SINTP(X,Y,N,X1,Y1)
      DIMENSION X(500),Y(500)
C
      DO 1  I=1,N
      K = I
      IF (X1.GT.X(I)) GO TO 1
      IF (X1.EQ.X(I)) GO TO 2
      IF (X1.LT.X(I)) GO TO 3
    1 CONTINUE
    2 Y1 = Y(K)
      GO TO 100
    3 IF (K.EQ.1) K=2
      IF (K.EQ.N) K=N-1
      IF(Y(K-1).NE.Y(K)) GO TO 5
      Y1=Y(K)
      RETURN
    5 CONTINUE
      W1 = (X1-X(K)) *(X1-X(K+1))/(X(K-1)-X(K))/(X(K-1)-X(K+1))
      W2 = (X1-X(K-1))*(X1-X(K+1))/(X(K)-X(K-1))/(X(K)-X(K+1))
      W3 = (X1-X(K-1))*(X1-X(K))/(X(K+1)-X(K-1))/(X(K+1)-X(K))
      Y1 = Y(K-1)*W1+Y(K)*W2+Y(K+1)*W3
  100 RETURN
      END
```

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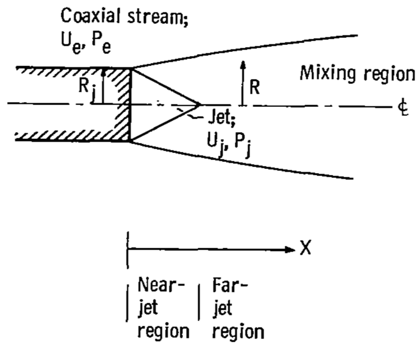


Figure 1. - Model.

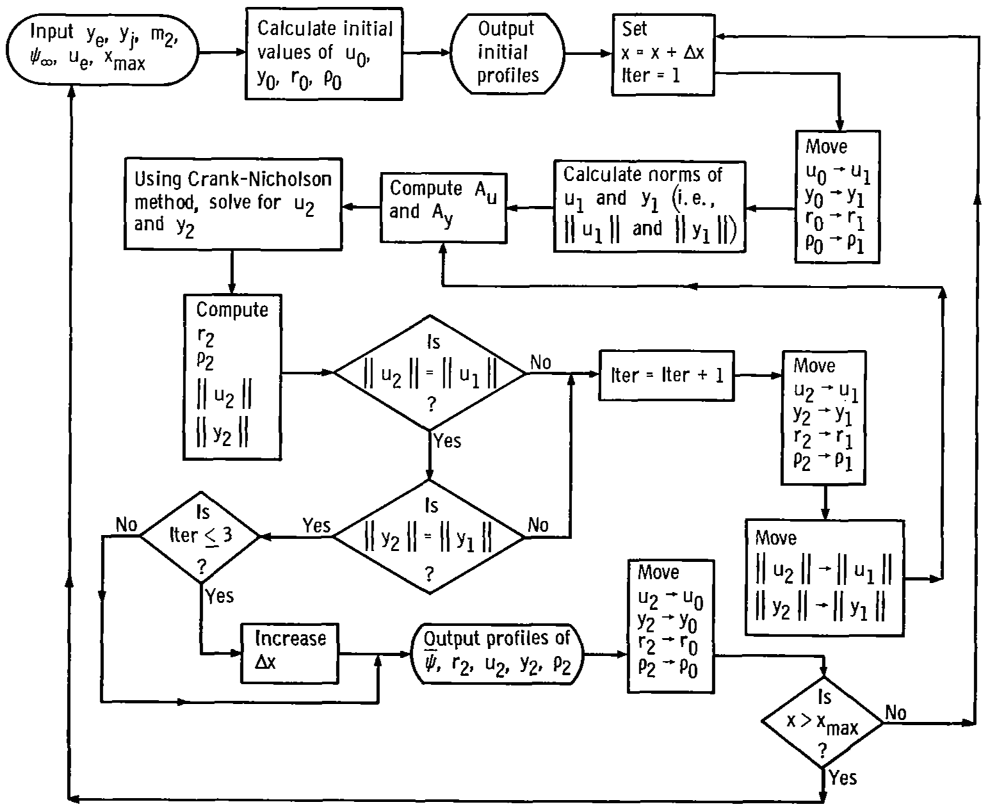
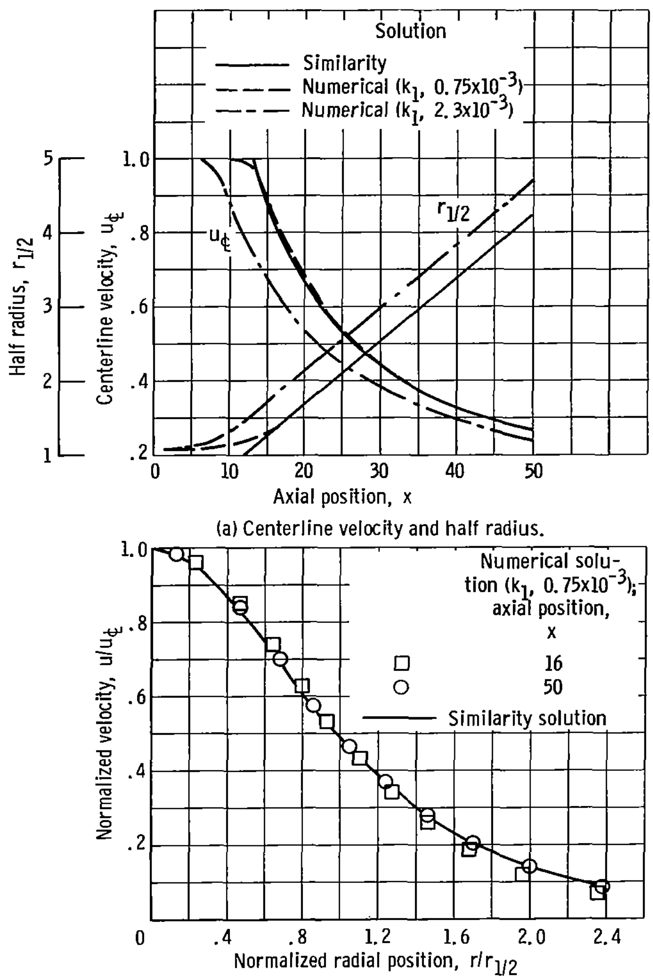


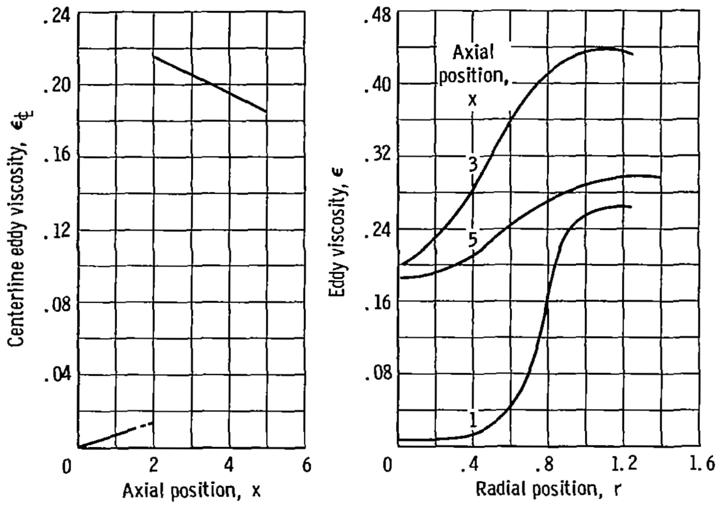
Figure 2. - General flow diagram for numerical solution. Subscripts 0, 1, and 2 denote values at $x - \Delta x$, initial values at x, and computed values at x, respectively.



(a) Centerline velocity and half radius.

(b) Velocity profile.

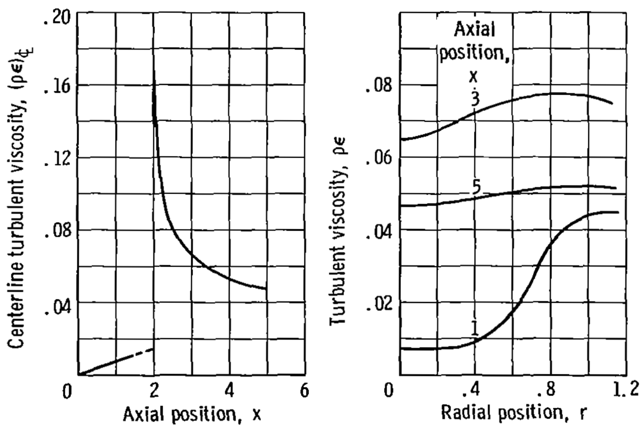
Figure 3. - Comparison of numerical and similarity solutions. Quiescent coaxial stream; constant density.



(a) Axial variation.

(b) Radial variation.

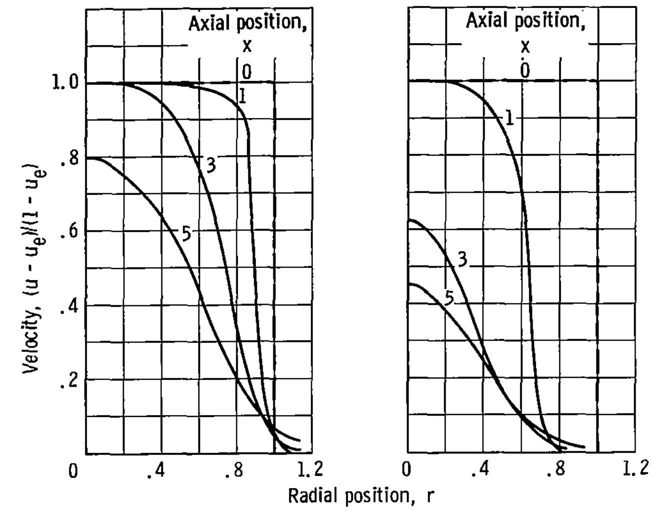
Figure 4. - Typical variation of eddy viscosity. Velocity ratio, 30; density ratio, 0.170; reference density, average of centerline density and coaxial-stream density.



(a) Axial variation.

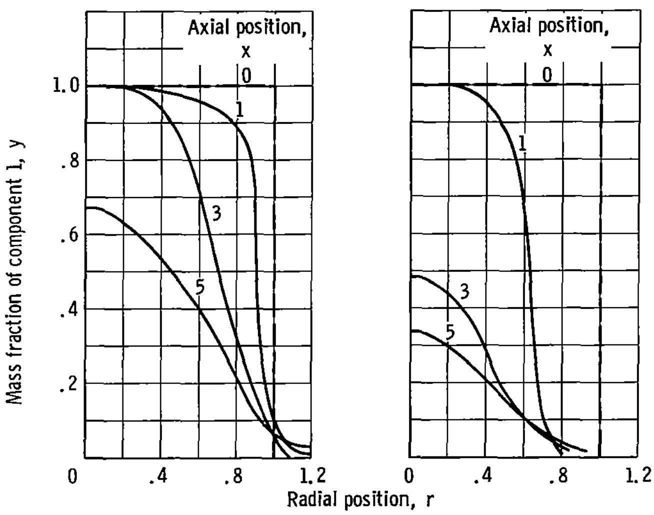
(b) Radial variation.

Figure 5. - Typical variation of product of density and eddy viscosity. Velocity ratio, 30; density ratio, 0.170; reference density, average of centerline density and coaxial-stream density.



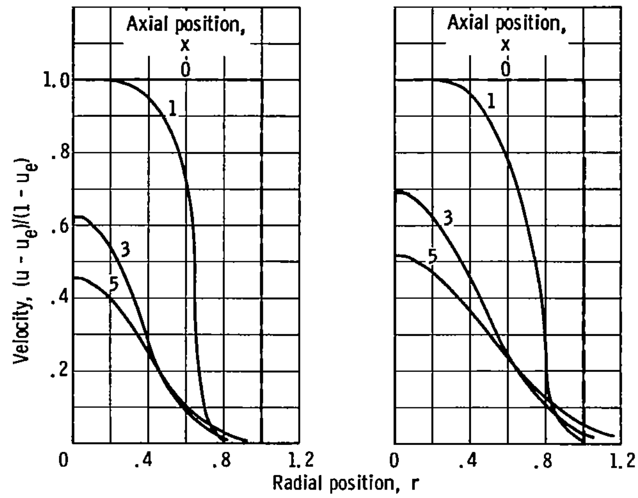
(a) Velocity ratio, 10. (b) Velocity ratio, 30.

Figure 6. - Effect of initial velocity ratio on velocity profiles. Density ratio, 0.364; reference density, average of centerline density and coaxial-stream density.



(a) Velocity ratio, 10. (b) Velocity ratio, 30.

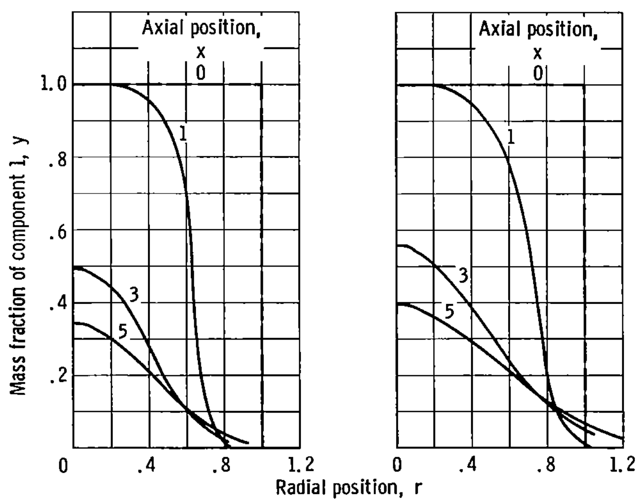
Figure 7. - Effect of initial velocity ratio on mass fraction profiles. Density ratio, 0.364; reference density, average of centerline density and coaxial-stream density.



(a) Density ratio, 0.364.

(b) Density ratio, 0.170.

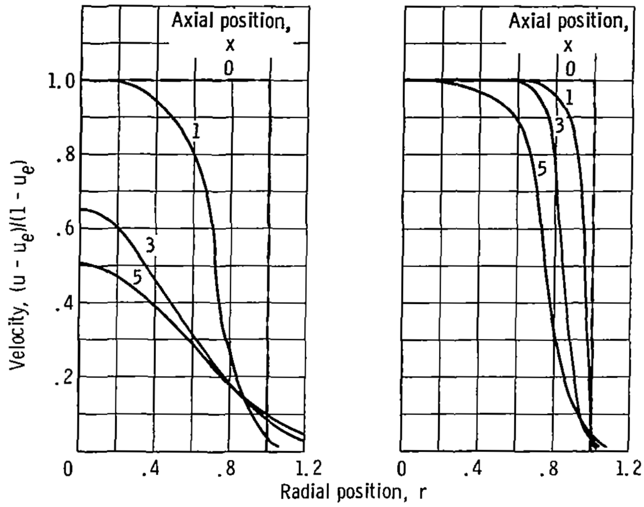
Figure 8. - Effect of initial density ratio on velocity profiles. Velocity ratio, 30; reference density, average of centerline density and coaxial-stream density.



(a) Density ratio, 0.364.

(b) Density ratio, 0.170.

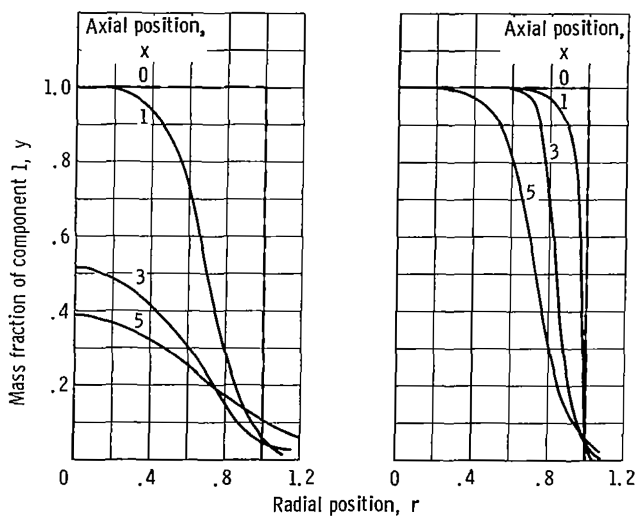
Figure 9. - Effect of initial density ratio on mass fraction profiles. Velocity ratio, 30; reference density, average of centerline density and coaxial-stream density.



(a) Reference density, centerline density.

(b) Reference density, coaxial-stream density.

Figure 10. - Effect of reference density on velocity profiles. Velocity ratio, 20; density ratio, 0.182.



(a) Reference density, centerline density.

(b) Reference density, coaxial-stream density.

Figure 11. - Effect of reference density on mass fraction profiles. Velocity ratio, 20; density ratio, 0.182.

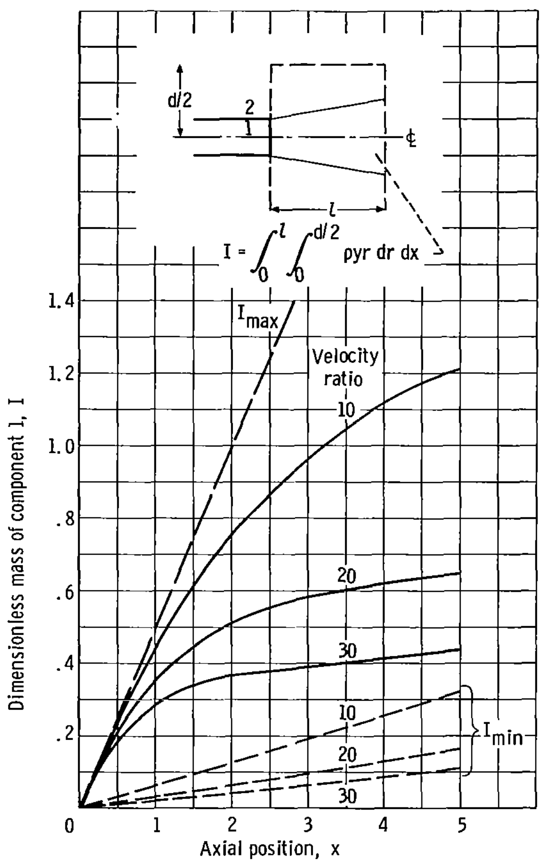


Figure 12. - Effect of initial velocity ratio on dimensionless mass of component 1. Density ratio, 0.364; reference density, average of centerline density and coaxial-stream density; ratio of reactor radius to jet radius, 4.

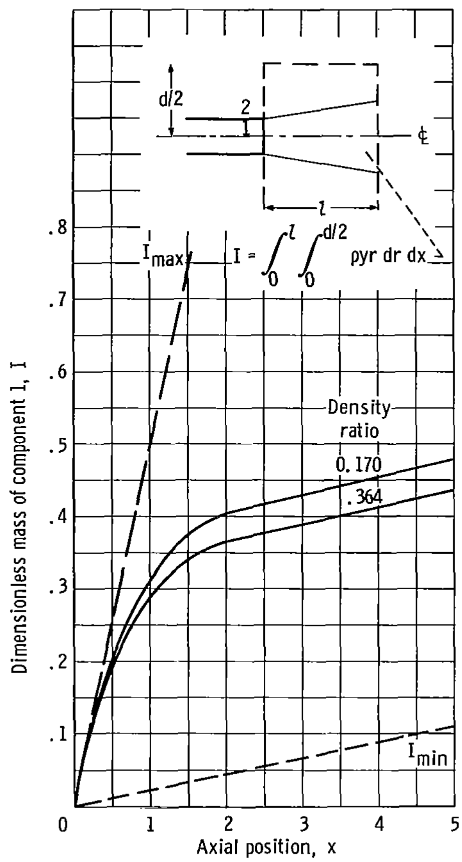


Figure 13. - Effect of initial density ratio on dimensionless mass of component 1. Velocity ratio, 30; reference density, average of centerline density and coaxial-stream density; ratio of reactor radius to jet radius, 4.

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